

***AN INFORMAL* INTRODUCTION TO FORMAL METHODS FOR SOFTWARE ENGINEERING**

May 26, 2020

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AGENDA



Software Verification

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Formal Methods

Formal Specification

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Formal Methods in Software Engineering

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Practice time!

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Practice time!

Take Home Messages

SOFTWARE VERIFICATION

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WHAT IT'S ALL ABOUT



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The process of checking that a system meets certain requirements derived from a given *specification*.

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- ▶ Malfunctions may cause financial losses.

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Why should we care?

- ▶ Computer systems are everywhere and we depend more and more on them;
- ▶ Malfunctions may cause financial losses **or worse!**

SOFTWARE VERIFICATION

CLASSIC TECHNIQUES



► Software Testing

SOFTWARE VERIFICATION

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 - ▶ dynamic analysis (software execution involved);

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SOFTWARE VERIFICATION

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 - ▶ static analysis (no software execution involved);

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 - ▶ dynamic analysis (software execution involved);
 - ▶ a suite of *test cases*, each specifying inputs and expected system behaviour, is typically produced by software testers.
- ▶ Code inspection
 - ▶ static analysis (no software execution involved);
 - ▶ careful scrutiny of the source code carried on by software engineers.

SOFTWARE VERIFICATION

WHEN CLASSIC TECHNIQUES FALL SHORT



Testing and code inspection are **very** effective at detecting bugs.

SOFTWARE VERIFICATION

WHEN CLASSIC TECHNIQUES FALL SHORT



Testing and code inspection are **very** effective at detecting bugs, but...

- ▶ cannot prove their absence;

[...] program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence.

– The humble programmer, E. W. Dijkstra [Dij72]

SOFTWARE VERIFICATION

WHEN CLASSIC TECHNIQUES FALL SHORT



Testing and code inspection are **very** effective at detecting bugs, but...

- ▶ cannot prove their absence;
- ▶ ineffective with concurrent systems;

[...] a concurrent program can withstand very careful scrutiny without revealing its errors. The only way we can be sure that a concurrent program does what we think it does is to prove rigorously that it does it.

– *Proving liveness properties of concurrent programs*, L. Lamport [LO82]

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Testing and code inspection are **very** effective at detecting bugs, but...

- ▶ cannot prove their absence;
- ▶ ineffective with concurrent systems;
- ▶ expensive and time-consuming.
- ▶ only feasible in later stages of the software lifecycle;



SOFTWARE VERIFICATION

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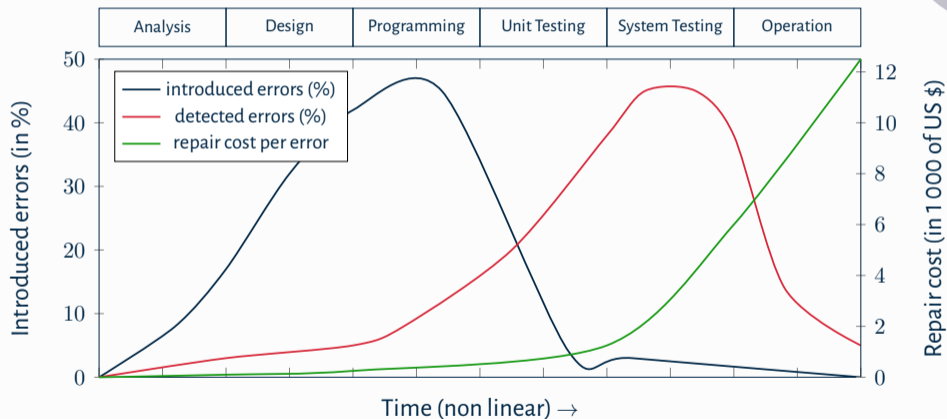


Figure: Error introduction, detection, and repair costs [BK08]

FORMAL METHODS

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 - ▶ system modelling languages;

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- ▶ Formal Verification
 - ▶ deductive verification (theorem proving);

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FORMAL METHODS



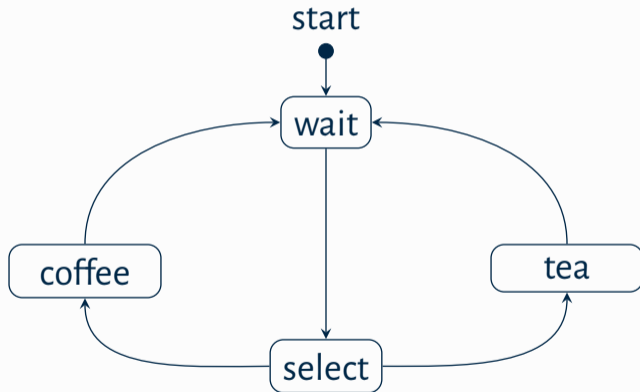
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- ▶ Formal Specification
 - ▶ system modelling languages;
 - ▶ property specification languages.
- ▶ Formal Verification
 - ▶ deductive verification (theorem proving);
 - ▶ automatic verification (model checking).
- ▶ Others (formal synthesis)

FORMAL SPECIFICATION: MODELS

TRANSITION SYSTEMS (TS)



- ▶ the set of states is called *state space*.

FORMAL SPECIFICATION: MODELS

MODELLING LANGUAGES: FEATURES



- ▶ Precise and Unambiguous;

FORMAL SPECIFICATION: MODELS

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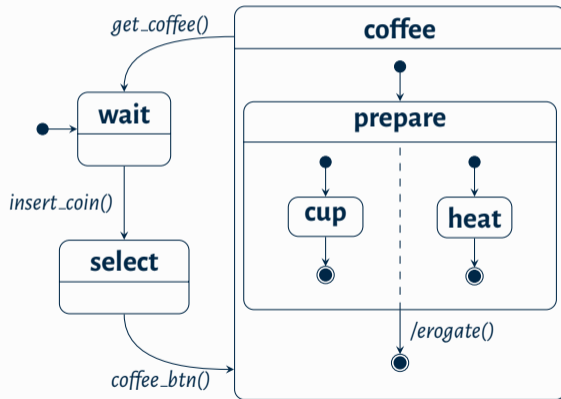
- ▶ Precise and Unambiguous;
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- ▶ “As simple as possible, as rich as needed” [Gli]
 - ▶ describe relevant aspects in a “natural” way;
 - ▶ trade-off between expressivity and analysis complexity;
 - ▶ using TS to model complex systems may be a bad idea: often higher-level languages are used instead.

FORMAL SPECIFICATION: MODELS

HIGHER-LEVEL MODELLING LANGUAGES: EXAMPLES



► Statecharts;

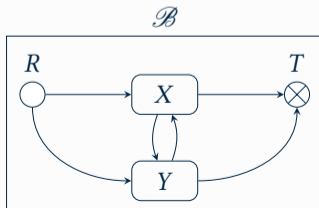
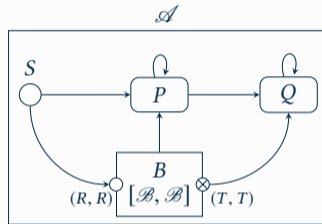


FORMAL SPECIFICATION: MODELS

HIGHER-LEVEL MODELLING LANGUAGES: EXAMPLES



- ▶ Statecharts;
- ▶ Hierarchical Machines;

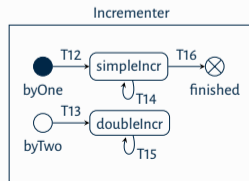
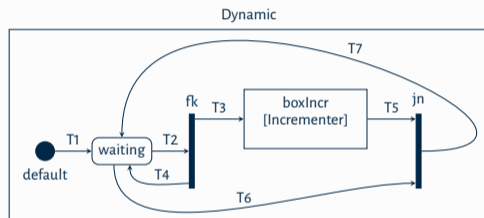




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```
active proctype A() {
    do
        :: (1) -> a=0;
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FORMAL SPECIFICATION: MODELS

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¹see Harel et al., [Har87; HN96]

²see Alur et al., [AKY99]

³see Benerecetti et al., [Ben+17]

⁴see [PRO]

FORMAL SPECIFICATION: MODELS

BEWARE OF MODEL REIFICATION!



All models are wrong, but some are useful.

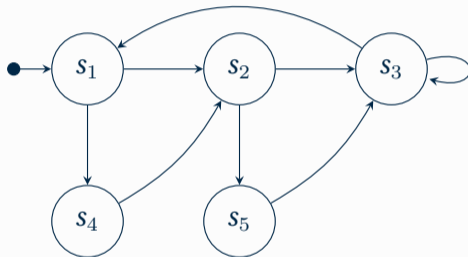
– G. Box [Box76]

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– T. Tarpey [Tar09]

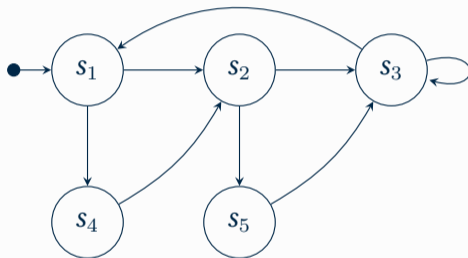
FORMAL SPECIFICATION: PROPERTIES

SYSTEM BEHAVIOURS



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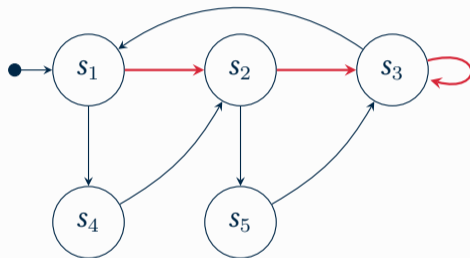


Possible behaviours:



FORMAL SPECIFICATION: PROPERTIES

SYSTEM BEHAVIOURS



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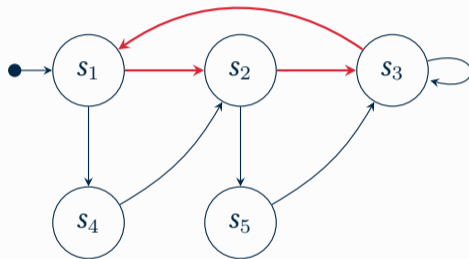
► $\pi_1 = S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_3 \rightarrow S_3 \rightarrow S_3 \rightarrow \dots$

$S_1 S_2 (S_3)^\omega$



FORMAL SPECIFICATION: PROPERTIES

SYSTEM BEHAVIOURS



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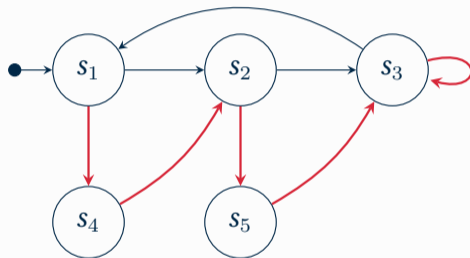
▶ $\pi_2 = S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow \dots$

$$(S_1 S_2 S_3)^\omega$$



FORMAL SPECIFICATION: PROPERTIES

SYSTEM BEHAVIOURS



Possible behaviours:

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$$\blacktriangleright \pi_3 = S_1 \rightarrow S_4 \rightarrow S_2 \rightarrow S_5 \rightarrow S_3 \rightarrow S_3 \rightarrow \dots$$

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FORMAL SPECIFICATION

TEMPORAL LOGICS: TIMELINE



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FORMAL SPECIFICATION

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- ▶ **LTL** (Linear-time Temporal Logic) was introduced by Pnueli in 1977 [Pnu77];

FORMAL SPECIFICATION

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- ▶ **CTL**, **CTL*** (Computation Tree Logic), a branching-time temporal logic;
- ▶ others (**CAReT** [AEM04], **HLTL**^ℓ, ...).

FORMAL SPECIFICATION

LTL SYNTAX



LTL extends propositional logic with temporal modalities.



FORMAL SPECIFICATION

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LTL syntax

LTL formulae over the set \mathcal{AP} of atomic proposition are formed according to the following grammar:

$$\phi ::= \top \mid a \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid X\phi \mid \phi_1 \mathbf{U} \phi_2 \mid \mathbf{F}\phi \mid \mathbf{G}\phi$$

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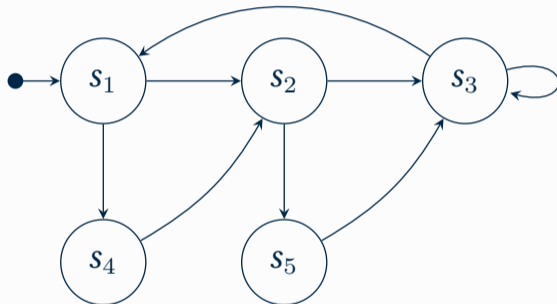
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LTL formulae are interpreted over system behaviours.

FORMAL SPECIFICATION

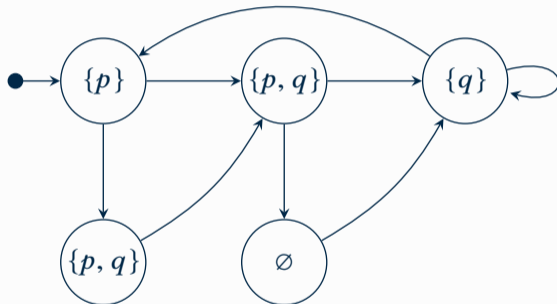
FROM TRANSITION SYSTEMS TO KRIPKE STRUCTURES





FORMAL SPECIFICATION

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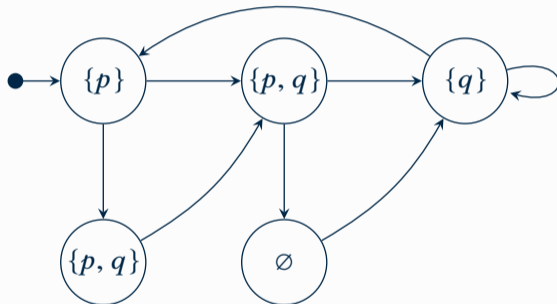


- ▶ we associate a set of atomic propositions to each TS state;



FORMAL SPECIFICATION

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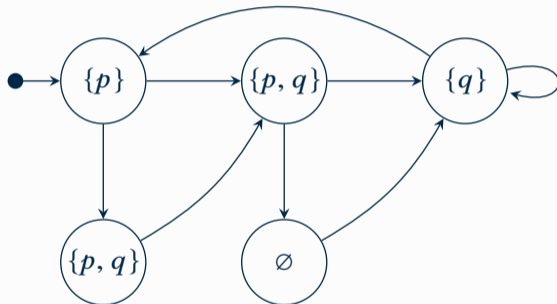


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FORMAL SPECIFICATION

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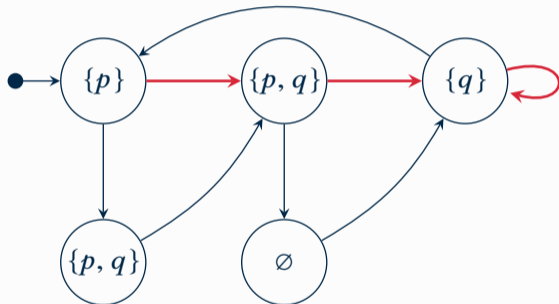


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FORMAL SPECIFICATION

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- ▶ $\pi_1 = \{p\} \rightarrow \{p, q\} \rightarrow \{q\} \rightarrow \{q\} \rightarrow \{q\} \rightarrow \{q\} \rightarrow \{q\} \rightarrow \dots$



FORMAL SPECIFICATION

LTL SEMANTICS – PART 1

LTL syntax

$$\phi ::= \top \mid a \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathbf{X}\phi \mid \phi_1 \mathbf{U} \phi_2 \mid \mathbf{F}\phi \mid \mathbf{G}\phi, \quad \text{with } a \in \mathcal{AP}.$$

Given a Kripke Structure behaviour $\pi = \pi_1 \rightarrow \pi_2 \rightarrow \dots$, with $\pi_i \in \wp(\mathcal{AP})$, and LTL formula ϕ , the *satisfaction* relation $\pi \models \phi$ is defined inductively as follows:



FORMAL SPECIFICATION

LTL SEMANTICS – PART 1

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FORMAL SPECIFICATION

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- ▶ $\pi \models \phi_1 \wedge \phi_2$ iff $\pi \models \phi_1$ and $\pi \models \phi_2$;



FORMAL SPECIFICATION

LTL SEMANTICS – PART 2

LTL syntax

$\phi ::= \top \mid a \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathbf{X}\phi \mid \phi_1 \mathbf{U} \phi_2 \mid \mathbf{F}\phi \mid \mathbf{G}\phi$, with $a \in \mathcal{AP}$.

- ▶ $\pi \models \mathbf{X}\phi$ iff ϕ holds in the **next** moment in time;





FORMAL SPECIFICATION

LTL SEMANTICS – PART 2

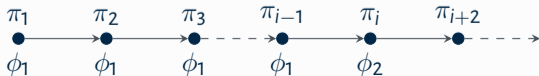
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- ▶ $\pi \models X\phi$ iff ϕ holds in the **next** moment in time;



- ▶ $\pi \models \phi_1 \mathbf{U} \phi_2$ iff ϕ_2 holds in a future moment, and ϕ_1 is true **until** ϕ_2 holds;





FORMAL SPECIFICATION

LTL SEMANTICS – PART 3

LTL syntax

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- $\pi \models \mathbf{F}\phi$ iff ϕ **finally** holds sometime in the future;





FORMAL SPECIFICATION

LTL SEMANTICS – PART 3

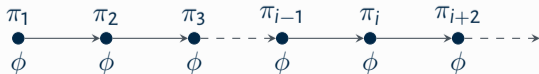
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- ▶ $\pi \models \mathbf{F}\phi$ iff ϕ **finally** holds sometime in the future;



- ▶ $\pi \models \mathbf{G}\phi$ iff ϕ holds **globally** (now and in every future moment);



THE LTL MODEL CHECKING PROBLEM



Given a Kripke Structure \mathcal{M} and an LTL formula ϕ , we say that

$$\mathcal{M} \models \phi$$

iff $\pi \models \phi$, for each behaviour π of \mathcal{M} .



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LTL Model Checking

The Model Checking problem amounts to decide whether $\mathcal{M} \models \phi$.

THE LTL MODEL CHECKING PROBLEM

SOME EXAMPLES



18

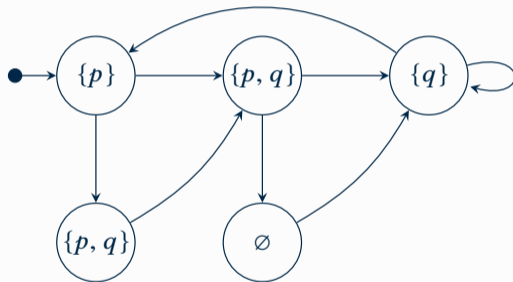


Figure: The Kripke Structure \mathcal{M}



THE LTL MODEL CHECKING PROBLEM

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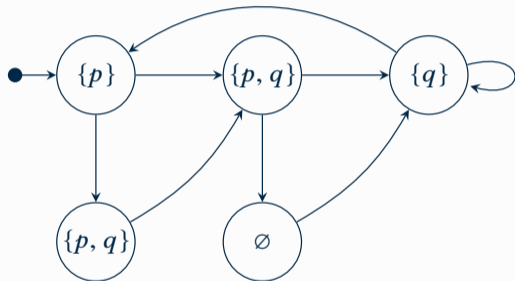


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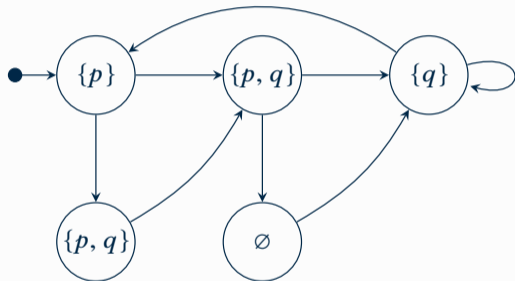


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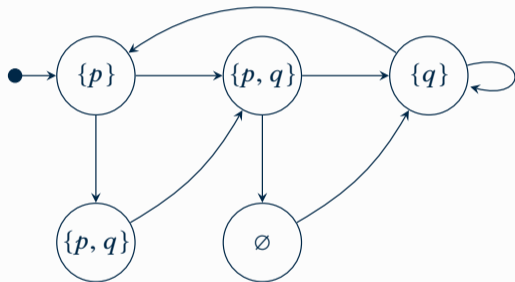


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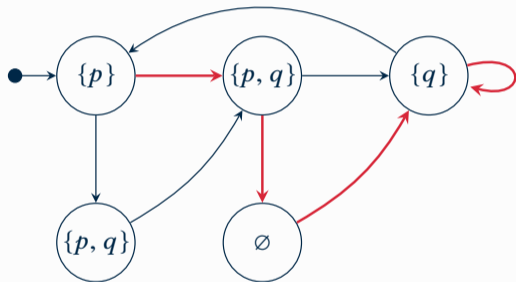


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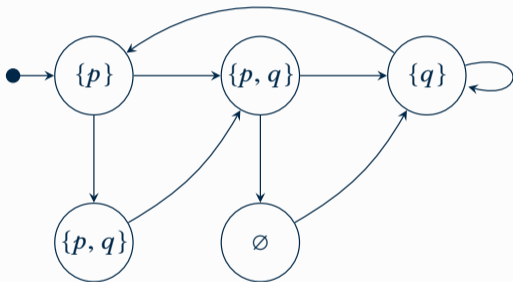


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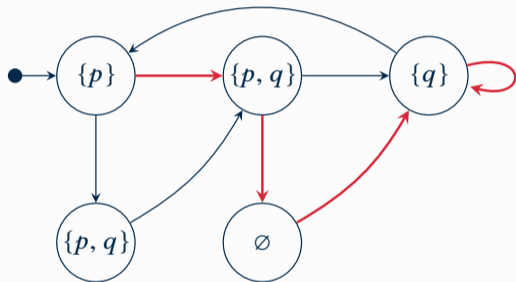


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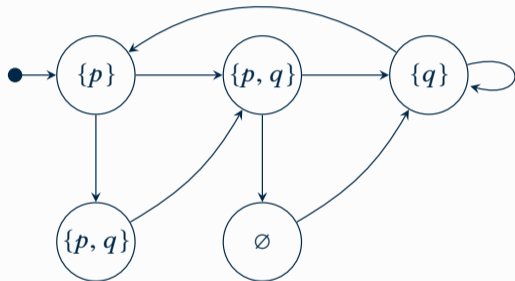


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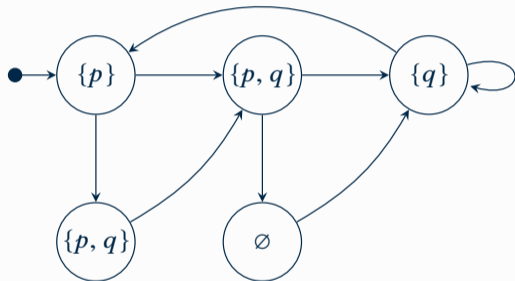


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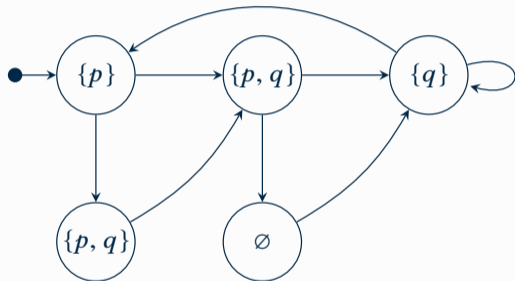


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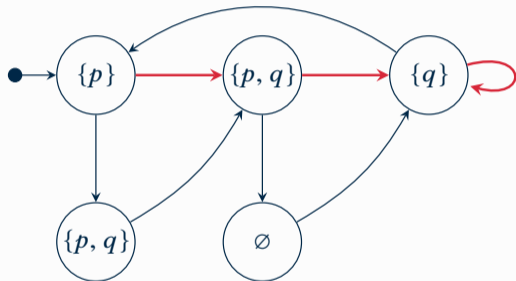
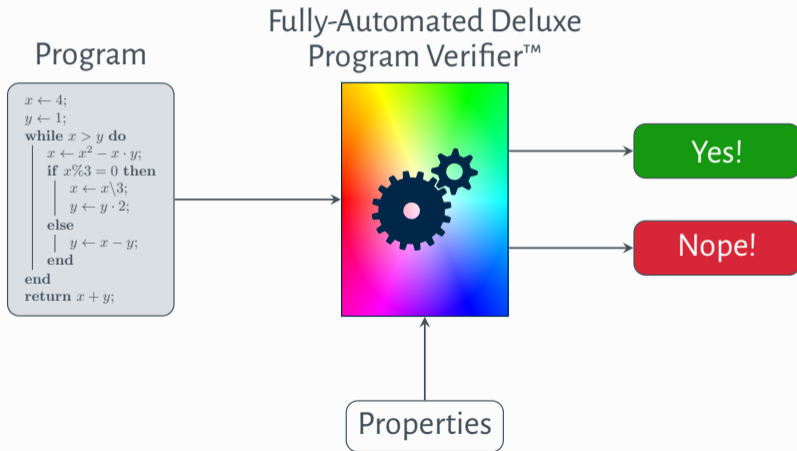


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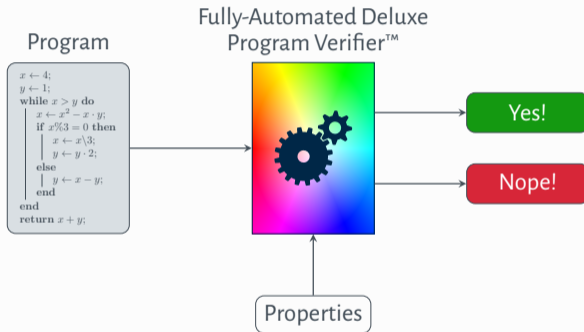
THE DREAM OF AUTOMATIC VERIFICATION





THE DREAM OF AUTOMATIC VERIFICATION

ACHIEVABLE?

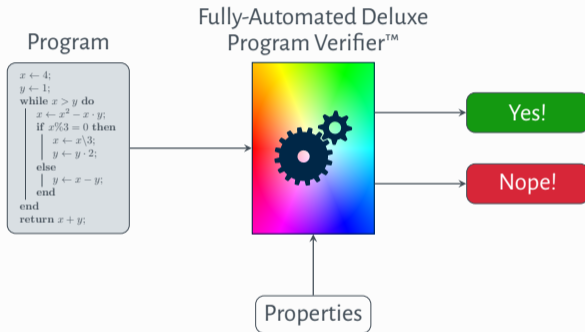


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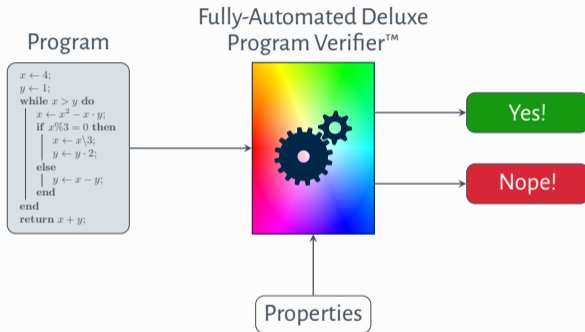


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- ▶ perhaps other interesting properties are decidable? **Bad news...**

THE FUNDAMENTAL LIMIT

UNDECIDABILITY



Rice's theorem [RVG]

Every non-trivial semantic property of programs is undecidable.

- ▶ a property is non-trivial if it neither is true for every program nor it's false for every program;
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An example

The property of returning 0 for some input is undecidable by Rice's Theorem.

THE FUNDAMENTAL LIMIT

UNDECIDABILITY



Implicit in Rice's Theorem is an **idealized** program model.

- ▶ Turing Machines have unbounded memory;
- ▶ A variable in Martin Davis' \mathcal{S} programs can be incremented indefinitely and never overflows;



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Concrete computing devices have **bounded** resources!

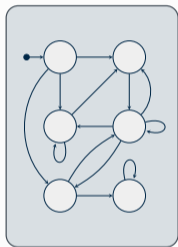
The model checking problem is decidable if we restrict ourselves to finite-state models.



AUTOMATIC VERIFICATION

MODEL CHECKERS

Finite State Model \mathcal{M}



Model Checker



$\mathcal{M} \models \text{Properties}$

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+counter-example

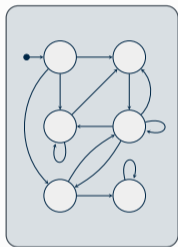
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AUTOMATIC VERIFICATION

MODEL CHECKERS

Finite State Model \mathcal{M}



Model Checker



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+counter-example

Properties (e.g. LTL)

Some well-known model checkers are [SPIN], [nuSMV], [TLC], [JPF].

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- ▶ 10 double variables (64 bit each) yield $2^{10 \times 64} \approx 10^{192}$ states;
- ▶ optimistic limit for a model checker? 10^{100} states [Kwo00].

FORMAL METHODS IN SOFTWARE ENGINEERING

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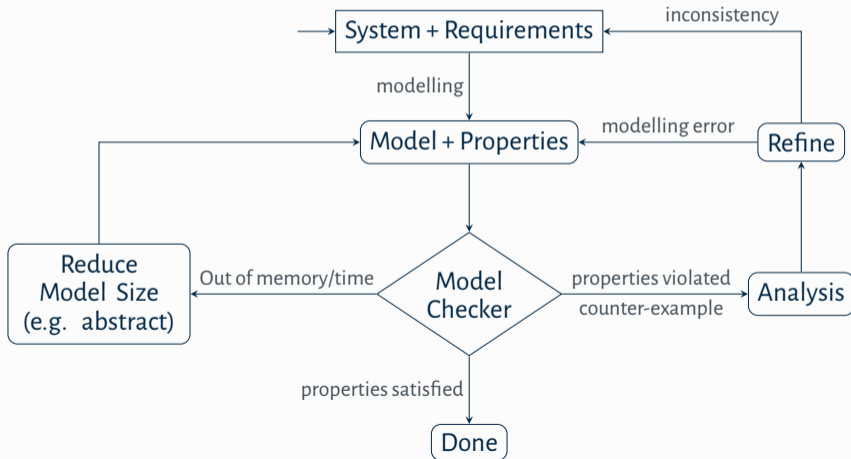
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- ▶ During Verification, FM can:
 - ▶ increase the confidence on system reliability;
 - ▶ help with traditional verification techniques (e.g. test case generation).



THE MODEL CHECKING PROCESS



A SUCCESS STORY

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 - ▶ Formal Methods and Model Checking (using TLC).

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- ▶ In two week, they learned how to use TLA+/TLC and wrote a detailed specification;
- ▶ Model-checked the specification using 10 EC2 instances, each with 8 cores plus hyperthreads, and 23 GB of RAM;
- ▶ Found a data-loss bug if a particular sequence of failures and recovery steps was interleaved with other processing; the shortest error trace exhibiting the bug contained 35 high-level steps.



A SUCCESS STORY

FORMAL METHODS AT AMAZON WEB SERVICES – PART 3

- ▶ This success led to management advocating TLA+ to other teams working on other products;

Product	Component	Benefits
DynamoDB	Replication & group-membership system	Found 3 bugs.
S3	Fault-tolerant low-level network algorithm	Found 2 bugs. Found further bugs in proposed optimizations.
	Background redistribution of data	Found 1 bug, and found a bug in the first proposed fix.
EBS	Volume management	Found 3 bugs.

Table: Benefits of using Formal Methods on different products at AWS

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- ▶ As with any tool, a model checker may contain software defects!

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- ▶ It can be easily integrated in existing development methodologies;
- ▶ It provides useful diagnostic counter-examples in case a property is violated;

PRACTICE TIME!



A CONCURRENT PROGRAM

```
process P0 {  
  while(true){  
    // noncritical section  
    flag_0 = 1;  
    while (flag_1) {}  
    // critical section  
    flag_0 = 0;  
    // noncritical section  
  }  
}
```

```
process P1 {  
  while(true){  
    // noncritical section  
    flag_1 = 1;  
    while (flag_0) {}  
    // critical section  
    flag_1 = 0;  
    // noncritical section  
  }  
}
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A CONCURRENT PROGRAM MODELLING

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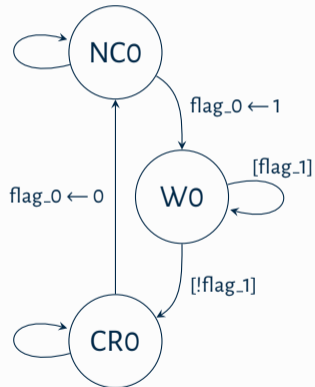


Figure: Model for process P0



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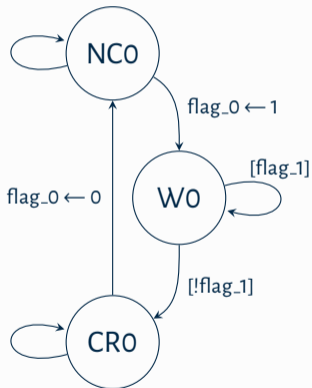


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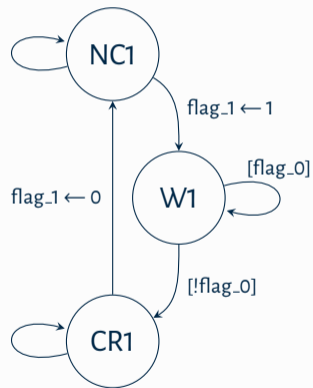


Figure: Model for process P1



A CONCURRENT PROGRAM

MODELLING: PARALLEL COMPOSITION

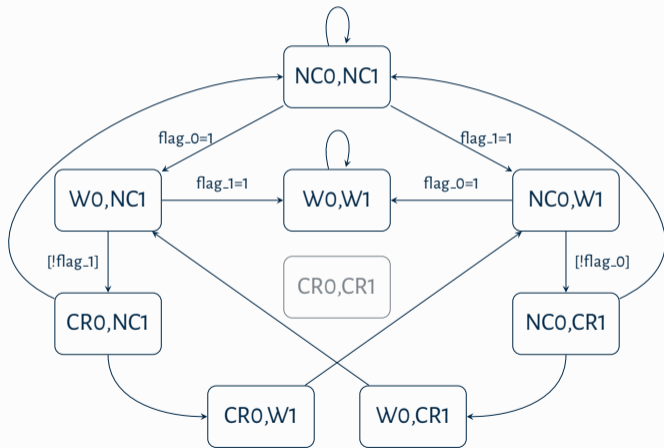


Figure: Asynchronous parallel composition of P_0 and P_1

Demo time
Model Checking with NuSMV

TAKE HOME MESSAGES

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 - ▶ System Specification (Transition Systems, higher-level specification languages);
 - ▶ Property Specification (LTL);
 - ▶ System Verification (Model Checking);
- ▶ Using Formal Methods;

You still there?

Any questions?



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